WESLEY COLLEGE

By daring & by doing

2018 YEAR 12 MATHEMATICS METHODS

Logarithms & Differentiation Applications

Test 2

Name: Solutions.

Marks: /50

Calculator Free (22 marks)

1. [2, 2 = 4 marks]

The displacement for an object is given by $x = e^{2t}\sin(t)$, where x is in metres and t is in seconds.

a) Determine the velocity equation.

$$\dot{x} = 2e^{2t}\sin(t) + e^{2t}\cos(t)$$

b) Show that when the object is at rest, $\tan(t) = -\frac{1}{2}$

Rest $\Rightarrow$ $\dot{x} = 0$

ie $2e^{2t}\sin(t) + e^{2t}\cos(t) = 0$

ie $e^{2t}(2\sin(t) + \cos(t)) = 0$

ie $2\sin(t) + \cos(t) = 0$

$2\tan(t) + 1 = 0$

$\tan(t) = \frac{-1}{2}$

/4

2. [2, 3, 2 = 7 marks]

Given the function: $y = x^4 + 4x^3 - 16x + 3$.

a) The gradient function, $y' = 4x^3 + 12x^2 - 16$ can be factorised as $y' = 4(x+2)^2(x-1)$ By using relevant tests to justify your answers, find and state the nature of all stationary points.

$y' = 4x^3 + 12x^2 - 16 = 4(x+2)^2(x-1)$

$y'' = 12x^2 + 24x$

Stationary pts: $y' = 0 \Rightarrow 4(x+2)^2(x-1) = 0$

$x = 1, -2$

$y''(1) > 0 \therefore$ min at $(1, -8)$

$y''(-2) = 0$ So use sign test.

x	-3	-2	-1
y'	$-$	0	$-$

$\therefore$ Horizontal P.O.I. at $(-2, 19)$

b) Using relevant tests to justify your answers, find and state all points of inflection.

POF $y'' = 0$

ie $12x(x+2) = 0$

$x = 0, -2$

from a) Horizontal POI at $(-2, 19)$

for $(0, 3)$

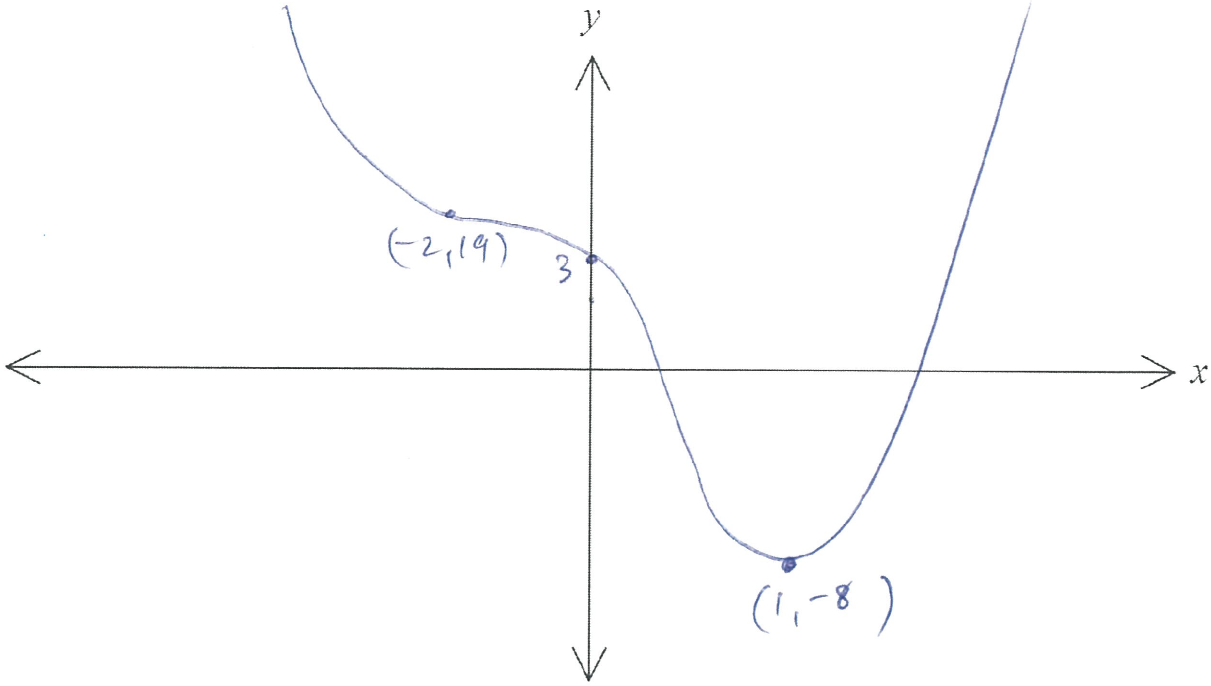
sign test

x	-1	0	$\frac{1}{2}$
y'	$-$	$-$	$-$

$\therefore$ POI at $(0, 3)$

$\frac{\quad}{7}$

c) Sketch the graph of $y = f(x)$, showing all important features.



3. [3 marks]

Given that $y = \sqrt{x}$, use the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$ to determine an approximate value for $\sqrt{50}$.

$y = x^{\frac{1}{2}}$ $\qquad x = 49 \Rightarrow \delta x = 1$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\delta y \approx \frac{1}{2\sqrt{49}} \times 1$

$= \frac{1}{14}$

$\therefore$ approx is $y + \delta y = 7\frac{1}{14}$.

$\frac{}{3}$

4. [2, 2, 4 = 8 marks]

a) Evaluate $\log_x x^2 - 6\log_x y + 3\log_x (xy)^2$

$= 2\log_x x - 6\log_x y + 6\log_x x + 6\log_x y$

$= 2 + 6$

$= 8$

b) Given $2\log_n x - 1 = \log_n 25$, write x in terms of n.

ie $\log_n x^2 - \log_n n = \log_n 25$

ie $\log_n\left(\frac{x^2}{n}\right) = \log_n 25$

$\therefore \frac{x^2}{n} = 25 \Rightarrow x^2 = 25n$

$\therefore x = 5\sqrt{n}$ (omit $-5\sqrt{n}$)

c) If $5^x = 3$ and $5^y = 4$, express in terms of x and/or y:

(i) $\log_5 0.75 = \log_5 \frac{3}{4}$

$= \log_5 3 - \log_5 4$

$= x - y$

(ii) $\log_5 100 = \log_5 (4 \times 25)$

$= \log_5 4 + \log_5 25$

$= y + 2$

End of Part A

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Calculator Section **(28 marks)**

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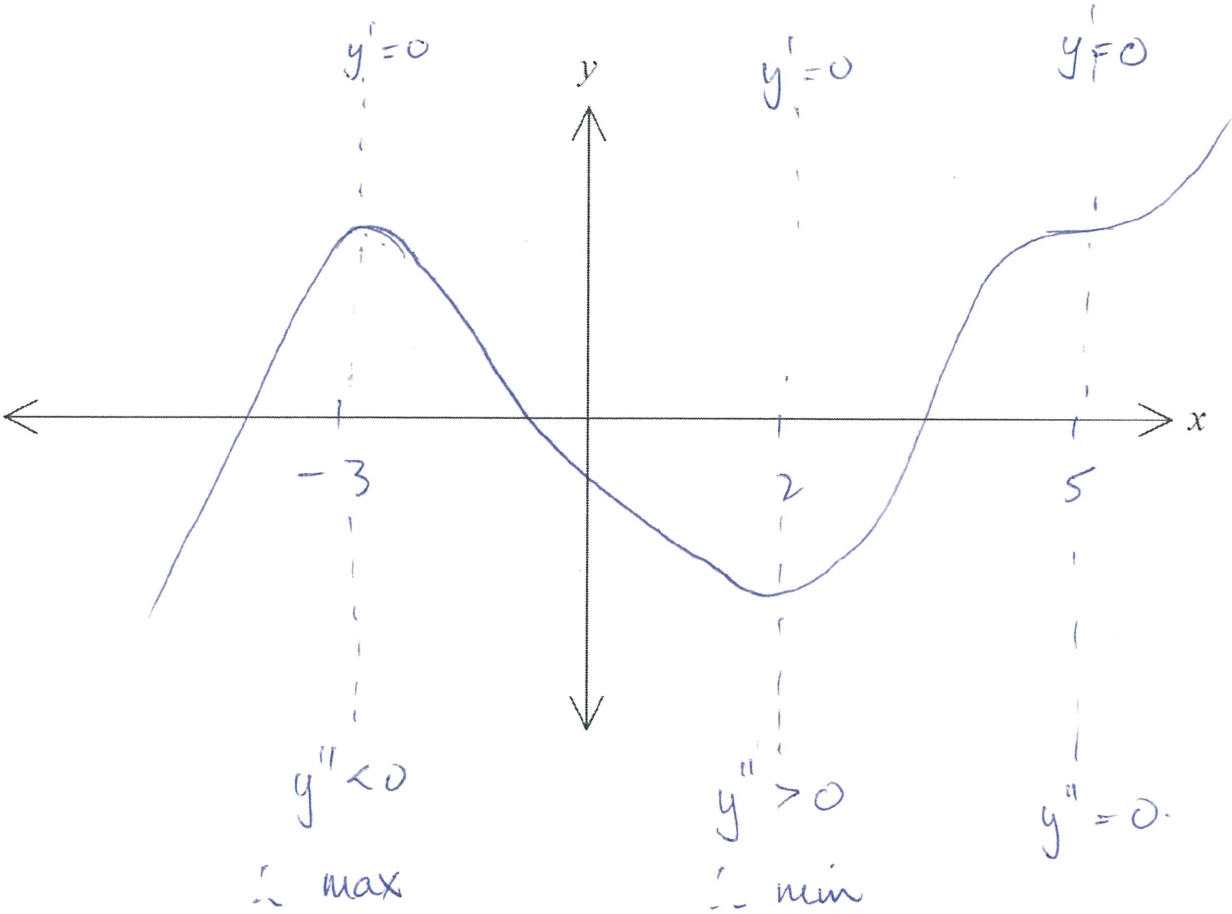
5. [4 marks]

Draw one possible function with all the following features:

$$\frac{dy}{dx}=0 \quad for \quad x=-3, \; x=2, \; x=5$$

$$\frac{d^2y}{dx^2}<0 \; for \; x=-3 \qquad \frac{d^2y}{dx^2}>0 \; for \; x=2 \; and \qquad \frac{d^2y}{dx^2}=0 \; for \; x=5$$



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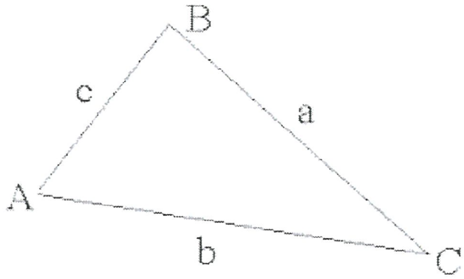
6. [3 marks]

The area of a triangle can be calculated by the formula $Area = \dfrac{ab\sin C}{2}$.



Using the incremental formula, determine the approximate change in area of an equilateral triangle, with each side 20 cm, when each side increases by 0.1 cm.

$a = b = x \qquad \angle C = \pi/3$

$A = \frac{1}{2}x^2 \sin(\pi/3)$

$= \frac{\sqrt{3}}{4}x^2$

$x = 20 \qquad \delta x = 0.1$

$\delta A \approx \frac{dA}{dx} \times \delta x$

$= \frac{2\sqrt{3}}{4}(20)0.1$

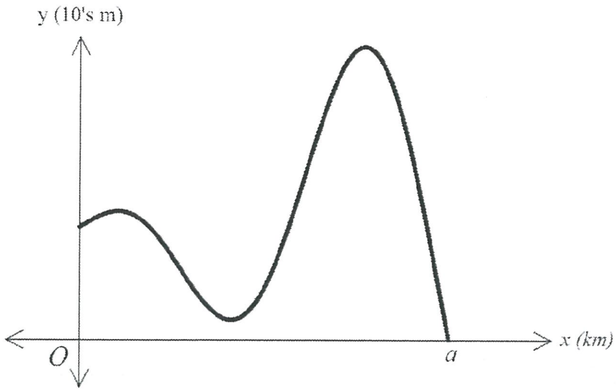
$= 1.732$

$\therefore$ approx change in area of $1.732\ cm^2$

$\frac{}{3}$

7. [1, 1, 3 = 5 marks]

The cross-section of the Darling Range, east of Perth is shown below.



The cross-sectional curve is given by $y = x\cos(x) + 4,\ 0 \le x \le a$.

a) Determine the value of a to two decimal places.

$a = 8.35$

b) Determine the height of the highest point on the range?

$y(6.437) = 10.36$

$\therefore$ highest point is 103.6 m

c) Moving away from O, down the far side of the range from the highest point, where is the steepest part of the hill side?

$y'' = -x\cos(x) - 2\sin(x) = 0$

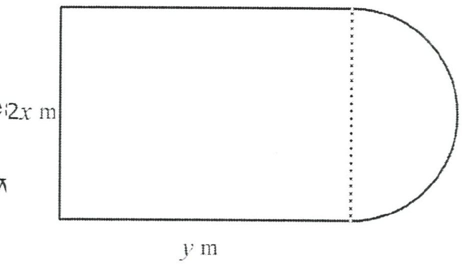
Solving $x = 8.096$, $y = 2.060$

So the steepest part is 8.1 km from O at a height of 20.6 m.

$\frac{}{5}$

8. [2, 1, 4 = 7 marks]

A 200 m fence is placed around a lawn which has the shape of a rectangle with a semi-circle on one of its side.



a) Using the dimensions on the diagram, clearly show that $y = 100 - x - \frac{\pi}{2}x$

$$2y + 2x + \pi x = 200$$

$$y + x + \frac{\pi}{2}x = 100$$

$$y = 100 - x - \frac{\pi}{2}x$$

b) Hence, determine the area of the lawn $A(x)$, in terms of x only.

$$A(x) = 2x\left(100 - x - \frac{\pi}{2}x\right) + \frac{\pi x^2}{2}$$

$$= -2x^2 + 200x - \frac{\pi x^2}{2}$$

c) Using calculus techniques determine the dimensions of the lawn if it has a maximum area and state this area.

$$\frac{dA}{dx} = -4x + 200 - \pi x \qquad \frac{d^2A}{dx^2} = -7.1 < 0 \therefore \text{max}$$

$$\frac{dA}{dx} = 0$$

ie $-4x + 200 - \pi x = 0$

$$x = \cancel{50}\ 28$$

So area is ~~5000 m²~~ 2800 m²

~~Dimensions are 100 m x 50 m~~

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9. [2, 2, 2, 3 = 9 marks]

A Cobalt projectile is travelling along a magnetic rail. It moves in a straight line such that its displacement from O, after t seconds is x metres where $x = t^3 - 10t^2 + 29t - 20$.

Determine, writing all relevant equations and giving answers correct to 2 decimal places:

a) the initial speed and acceleration of projection,

$$\dot{x}(0) = 29 \text{ m/s}$$
$$\ddot{x}(0) = -20 \text{ m/s}^2$$

b) when the particle is at rest,

$\dot{x} = 0$ ie $3t^2 - 20t + 29 = 0$.

$t = 2.13\text{s}, \; 4.54\text{s}$

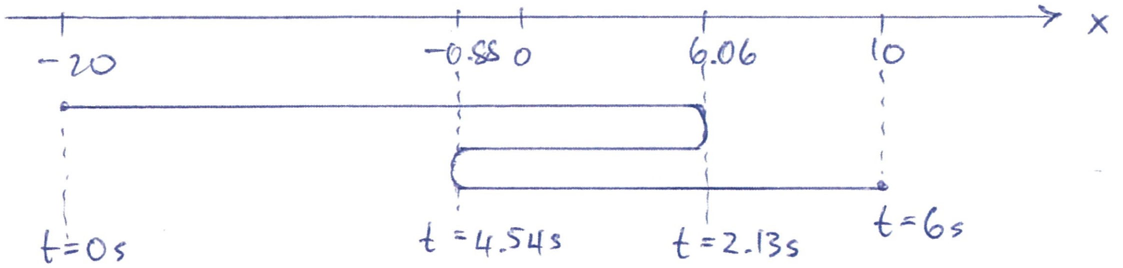
c) the particle's velocity when the acceleration is 0,

$\ddot{x} = 0$ ie $6t - 20 = 0$

$t = 3\frac{1}{3}$

$\dot{x}(3\frac{1}{3}) = -4\frac{1}{3}$ m/s $(-4.33$ m/s$)$

d) the total distance travelled in the first 6 seconds.



distance $= 26.06 + 6.94 + 10.88$

$= 43.88$ m $\quad \left(\int_0^6 |3t^2 - 20t + 29| \, dt \right)$

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End of Part B